Third Grade Unit Four
Operations and Algebraic Thinking:
Patterns in Addition and Multiplication
# Unit 4
Operations in Algebraic Thinking: Patterns in Addition and Multiplication

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UNIT OVERVIEW

In this unit, students will:

- Understand concepts of area and relate area to multiplication and addition.
- Find the area of a rectangle with whole-number side lengths by tiling it.
- Multiply side lengths to find areas of rectangles with whole-number side lengths in context of solving real world and mathematical problems.
- Construct and analyze area models with the same product.
- Describe and extend numeric patterns.
- Determine addition and multiplication patterns.
- Understand the commutative property’s relationship to area.
- Create arrays and area models to find different ways to decompose a product.
- Use arrays and area models to develop understanding of the distributive property.
- Solve problems involving one and two steps and represent these problems using equations with letters “n” or “x” representing the unknown quantity.
- Create and interpret pictographs and bar graphs.
- Find area of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts.

The understanding of and ability to use multiplication and division is the basis for all further mathematics work and its importance cannot be overemphasized. As students move through upper elementary grades and middle school, the foundation laid here will empower them to work with fractions, decimals, and percent.

Area is a measure of the space inside a region or how much it takes to cover a region. As with other attributes, students must first understand the attribute of area before measuring.

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column.

Using this model, students should be able to create arrays to solve real-life problems involving multiplication and apply this concept with addition, subtraction, and division to solve equations involving two steps or more to find the solution.

Adapted from NC Dept of Public Instruction, Teaching Resources

STANDARDS FOR MATHEMATICAL CONTENT
Solve problems involving the four operations, and identify and explain patterns in arithmetic.

**MCC.3.OA.8.** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**MCC.3.OA.9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Represent and interpret data.

**MCC.3.MD.3.** Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

**MCC.3.MD.4.** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

**Geometric Measurement: understand concepts of area and relate area to multiplication and to addition.**

**MCC.3.MD.5.** Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of $n$ square units.

**MCC.3.MD.6.** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
MCC.3.MD.7. Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show, in a concrete case, that the area of a rectangle with whole-number side lengths \(a\) and \(b + c\) is the sum of \(a \times b\) and \(a \times c\). Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real-world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.

Students are expected to:
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

****Mathematical Practices 1 and 6 should be evident in EVERY lesson. ***

ENDURING UNDERSTANDING

- Area models are related to addition and multiplication.
- Area covers a certain amount of space using square units.
- When finding the area of a rectangle, the dimensions represent the factors in a multiplication problem.
- Each dimension can be considered repeated addition.
- Multiplication is repeated addition.
- Multiplication can be used to find the area of rectangles with whole numbers.
• Area models of rectangles and squares are directly related to the commutative property of multiplication.

• Rearranging an area such as 24 sq. units based on its dimensions or factors does NOT change the amount of area being covered (Van de Walle, pg 234). Ex. A 3 X 8 is the same area as a 4 X 6, 2 X 12, and a 1 X 24.

• A product can have more than two factors.

• Area in measurement is equivalent to the product in multiplication.

• Area models can be used as a strategy for solving multiplication problems.

• Some word problems may require two or more operations to find the solution.

**ESSENTIAL QUESTIONS**

- By using an area model to learn multiplication, how many number patterns of multiplication are displayed?
- Can one area measurement of a rectangle produce different dimension measurements? Of a square?
- Can the same area measurement produce different size rectangles? (Ex. 24 sq.units can produce a rectangle that is a 3 x 8, 4 x 6, 1 x 24, 2 x 12)
- Can you find area without perimeter? Perimeter without area?
- Do different dimensions with the same area cover the same amount of space? (Ex. Is a 3 x 8 the same area as a 1 x 24?)
- How are multiplication and addition different? How are they the same?
- How are the same number of tiles with different square unit measurements such as square feet, inches, cm, and mm significantly different?
- How can an addition table help you explain the commutative property of multiplication?
- How can multiple math operations be used to solve real world problems?
- How can the same area model represent both multiplication and division?
- How can we connect multiplication facts with their area models?
- How can we determine numbers that are missing on a multiplication chart by knowing multiplication patterns?
- How can we use patterns to solve problems?
- How do estimation, multiplication, and division help us solve problems in everyday life?
- How do rectangle dimensions impact the area of the rectangle?
- How does an area model relate to the commutative property of multiplication?
- How does drawing an area model help us think about different ways to decompose a number?
- How does knowing the area of a square or rectangle relate to knowing different multiplication facts?
- How does knowing the dimensions of a rectangle relate to multiplication?
- How does knowing the dimensions of two sides help you determine the perimeter of the whole plane figure?
• How does the order of the digits in a multiplication problem affect the product?
• How does understanding the distributive property help us multiply large numbers?
• How is a pattern related to multiplication?
• How is the commutative property of multiplication evident in an area model?
• How is the decomposition of a factor in an equation related to the distributive property of multiplication?
• Is there more than one way of multiplying to get the same product?
• Why are mathematical expressions important in problems involving two or more math operations?
• What does it mean to decompose a number?
• What is area?
• What is the connection between area models and skip counting?
• What is the connection between a pictograph and problem solving?
• What is the relationship between a multiplication chart and an area model?
• What is the relationship between addition and multiplication?
• What is the relationship between area and perimeter?
• What is the relationship between dimensions and factors?
• What is the relationship between the product and the sum?
• What makes an area model a good representation for multiplication?
• How can what I understand about area help me to understand multiplication and addition patterns?
• What patterns of multiplication can we discover by studying a multiplication chart?
• What is a pattern?
• What is the difference between an expression and an equation?
• What is the relationship between a pictograph and problem solving?
• What’s the relationship between the picture’s value and patterns found in multiplication?
• When solving equations, why must the operations be completed in a certain order?
• Why are square units commonly associated with finding area?
• Why is it important to know the difference in between the square unit measurements?
• Why is it important to understand that more than one math operation may be needed to solve a problem?

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

• Addition
• Skip counting
• Multiplying
• Two-dimensional plane figures
• Area measures space inside a region
• Area can be determined using multiplication
• Square unit measurements
• Creating area models
• Understanding areas relationship to arrays
• Using area models and arrays to solve word problems
• Factors of products
• Dimensions of area models
• Decomposing
• Commutative Property of Multiplication
• Distributive Property of Multiplication
• Solving two-step word problems

**Selected Terms and Symbols**

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

Teachers should present the vocabulary to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

**Vocabulary and Symbols**

- addend
- addition
- area
- area model
- array
- attribute
- bar graph
- centimeter
- commutative property of multiplication
- decomposing
- difference
- dimensions
- distributive property of multiplication
- divide
- equation
- estimation
- expression
- factor
- gap
- inch
- line plot
- mental computation
Georgia Department of Education  
Common Core Georgia Performance Standards Framework

Third Grade Mathematics • Unit 4

- multiply
- non-standard units
- operation
- plane figure
- product
- quotient
- rounding
- scale
- side length
- square unit
- subtract
- sum tiling
- unknown/variable

STRATEGIES FOR TEACHING AND LEARNING
Adapted from North Carolina Dept. of Public Instruction Teaching Resources

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Students gain a full understanding of which operation to use in any given situation through contextual problems. Number skills and concepts are developed as students solve problems. Problems should be presented on a regular basis as students work with numbers and computations.

Researchers and mathematics educators advise against providing “key words” for students to look for in problem situations because they can be misleading. Students should use various strategies to solve problems. Students should analyze the structure of the problem to make sense of it. They should think through the problem and the meaning of the answer before attempting to solve it.

Encourage students to represent the problem situation in a drawing or with counters or blocks. Students should determine the reasonableness of the solution to all problems using mental computations and estimation strategies.

Students can use base–ten blocks on centimeter grid paper to construct rectangular arrays to represent problems.

Students are to identify arithmetic patterns and explain these patterns using properties of operations. They can explore patterns by determining likenesses, differences and changes. Use patterns in addition and multiplication tables.

Represent and interpret data.

Representation of a data set is extended from picture graphs and bar graphs with single-unit scales to scaled picture graphs and scaled bar graphs. Intervals for the graphs should relate to multiplication and division within 100 (product is 100 or less and numbers used in division are 100 or less). In picture graphs, use multiplication fact values, with which students are having difficulty, as the icons. For example, determine that the three icons represents 21 people. The intervals on the vertical scale in bar graphs should not exceed 100.

Students are to draw picture graphs in which a symbol or picture represents more than one object. Bar graphs are drawn with intervals greater than one. Ask questions that require students to compare

□ represents 7 people.
quantities and use mathematical concepts and skills. Use symbols on picture graphs that student can easily represent half of, or know how many half of the symbol represents.

Students are to measure lengths using rulers marked with halves and fourths of an inch and record the data on a line plot. The horizontal scale of the line plot is marked off in whole numbers, halves or fourths. Students can create rulers with appropriate markings and use the ruler to create the line plots

**Geometric measurement – understand concepts of area and relate area to multiplication and to addition.**

Students can cover rectangular shapes with tiles and count the number of units (tiles) to begin developing the idea that area is a measure of covering. Area describes the size of an object that is two-dimensional. The formulas should not be introduced before students discover the meaning of area.

The area of a rectangle can be determined by having students lay out unit squares and count how many square units it takes to completely cover the rectangle completely without overlaps or gaps. Students need to develop the meaning for computing the area of a rectangle. A connection needs to be made between the number of squares it takes to cover the rectangle and the dimensions of the rectangle. Ask questions such as:

- What does the length of a rectangle describe about the squares covering it?
- What does the width of a rectangle describe about the squares covering it?

The concept of multiplication can be related to the area of rectangles using arrays. Students need to discover that the length of one dimension of a rectangle tells how many squares are in each row of an array and the length of the other dimension of the rectangle tells how many squares are in each column. Ask questions about the dimensions if students do not make these discoveries. For example:

- How do the squares covering a rectangle compare to an array?
- How is multiplication used to count the number of objects in an array?

Students should also make the connection of the area of a rectangle to the area model used to represent multiplication. This connection justifies the formula for the area of a rectangle.

Provide students with the area of a rectangle (i.e., 42 square inches) and have them determine possible lengths and widths of the rectangle. Expect different lengths and widths such as 6 inches by 7 inches, or 3 inches by 14 inches.

**EVIDENCE OF LEARNING**

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Understand that area means to cover a certain amount of space without gaps.
- Discuss and demonstrate the relationship between area and multiplication.
- Discuss and demonstrate how area is repeated addition based on a rectangle’s two dimensions or factors.
- Understand the decomposition of an area model through multiplication and addition.
- Create different area models with the same area by using different dimensions or factors.
- When problem solving, find the missing dimensions or factors.
- Relate area to the commutative, identity, and distributive properties of multiplication.
• Identify number patterns and apply them in a multiplication table.
• Draw a scaled pictograph and bar graph and apply multiplication patterns based on the intervals given to each picture.

**TASKS**

The following tasks represent the level of depth, rigor, and complexity expected of all third grade students. These tasks or a task of similar depth and rigor should be used to demonstrate evidence of learning. It is important that all elements of a task be addressed throughout the learning process so that students understand what is expected of them. The following is a description of the types of tasks you will see in this unit and their purpose.
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Dr. John D. Barge, State School Superintendent
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SCAFFOLDING TASK: COVER ME!

Adapted from Teaching Student Centered Mathematics, by John A. Van de Walle, 2006 pg 235

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

b. A plane figure which can be covered without gaps or overlaps by \( n \) unit squares is said to have an area of \( n \) square units.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Misconception- Van de Walle states that area is a measure of the space inside a region or how much it takes to cover a region. As with other attributes, students must first understand the attribute of area before measuring. Data from the seventh National Assessment of Educational Progress suggest that fourth-and eighth-grade students have an incomplete understanding of area (Martin & Strutchens, 2000).

Many conclude that the only way to teach area is by using squares since they are very nice units and easy to use for covering. However, any tile that conveniently fills up a space can be used. Even filling a region with uniform circles or lima beans provides a useful idea of what it means to measure area (Teaching Student Centered Mathematics, Van de Walle, John A., p 262).

Background Knowledge- Students should know that a rectangle is a two-dimensional plane figure.

ESSENTIAL QUESTION

What is area?
MATERIALS

tangrams (a blackline master can be found on pg 18)
math journal/learning log,
Grandfather Tang’s Story by Ann Tompert,
unitedstreaming.com video entitled Using Tangrams

GROUPING

Partner

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will investigate area using tangrams. They will create different tangram pictures and discover that one set of tangrams can cover different shapes because it’s the same area. This task is a scaffold for one product having multiple multiplication sentences.

Comments

The lesson could open with the teacher giving each student three different pictures created out of square units. However, they would all have the same area! For example, if the area is 12 square units, they would each get a picture that is a 1 X 12, 2 X 6, and a 3 X 4. Next, have the students glue each picture in their learning log/math journal and write how they are the same and how they are different. This would be a great set-up for the unit that will follow.

Once completed, the teacher should allow time for sharing. Next, read the book Grandfather Tang’s Story. Following the reading of the story, you might show the students the two and a half minute video from unitedstreaming. Upon completion of this, begin a discussion about tangrams asking the following questions:

Discussion Questions:
What are tangrams? (Answer: A puzzle!)
What makes the puzzle unique? (Answer: It’s a rectangle cut into 7 shapes.)
After reading the book and looking at the mini video, what do you think is the puzzle’s relationship to math? (Broad answers which the teacher should record on chart paper so that she can go back and tie their thinking here with area later.)

Task Directions: Give each student a bag of tangrams, and have them explore. Alternatively, you may allow students to create their own set of tangrams using construction paper or card stock. Directions are available here:
http://www.heckscher.org/downloads/ED08_KidsCor_ActivityPages_Tangram.pdf

You may ask them to try to make different pictures/shapes. Allot a certain amount of time for this and then give them a sheet of white paper and have them trace around their favorite picture/shape. Once finished, the students stand up, take THEIR set of tangrams, and COVER someone else’s puzzle.
Once completed, the teacher will lead the students in the following discussion.

Discussion Questions:
Were all the pictures the same?
How did some of the pictures look?
What one unique trait did all of the pictures have in common? (This is important! Someone needs to respond that they used the same 7 shapes to cover different pictures!)

BIG QUESTION #1: What does this say about each picture? (The teacher should allow for deep reflection here. However, what she wants them to understand is although each picture is different, they are all using the same 7 pieces which were cut from the same size rectangle. )

BIG QUESTION #2: Are the pictures covering the same amount of space? (The teacher should be sure to hold up different pictures the students have created to drive this point home.)

Have the students go back to their math journal/learning log and examine the pictures they glued in their log at the beginning of the lesson closely. Tell them to think about the activity they just completed and examine each picture carefully and respond to the following questions.

FORMATIVE ASSESSMENT QUESTIONS
• What is the relationship between the pictures and the tangram activity?
• What is the relevance of studying this connection?

After listening to student responses and discussions, the teacher will introduce the vocabulary word area. She will go on to explain the definition and then ask them the following question: How does area relate to the tangram lesson and the pictures that were glued in your journals?

Once completed, the students can share their responses and the teacher will serve as facilitator during the discussion.

DIFFERENTIATION

Extension
• The teacher will give them different plane figures such as pentagons, hexagons, trapezoids, etc. and have them create their own tangram puzzle, design another picture, and have another student solve the puzzle using the pieces they created and vice-versa. However, the student who solved will then keep the pieces and create another picture and give the person back their pieces and have them solve the puzzle they created using their pieces.

Intervention
• Having the students work in small groups will provide support for students who struggle with this concept and will enable them to develop the ability to describe their thinking.
CONSTRUCTING TASK: FILL ‘ER UP!
Adapted from Teaching Student Centered Mathematics, by John A. Van de Walle pg. 236

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.
   b. A plane figure which can be covered without gaps or overlaps by \( n \) unit squares is said to have an area of \( n \) square units.

MCC3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

As the teacher, your objective in the beginning is to develop the idea that area is a measure of covering. Do not introduce formulas. Simply have the students fill the shapes and count the units. Be sure to include estimation before measuring (this is significantly more difficult than for length), use approximate language, and relate precision to the size of the units in the same manner as with length.

It is important to understand that filling regions with units and counting does little to help students develop multiplicative formulas. Even when rectangles are filled with a grid of squares, students are more likely to count the squares than to relate the number of squares to the dimensions of the rectangles (Van de Walle, page 236-237).

ESSENTIAL QUESTIONS

What is area?

MATERIALS

- markers
• circular disks
• color tiles
• lima beans
• math journal/learning logs

**GROUPING**

Partner

**TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

In this task, students will practice filling area and estimation. The teacher will begin with a review of the previous day’s lesson. Reinforcement will be given to the term area and relate area to real-life to make it relevant. This could include but is not limited to their homes, their classroom, neighborhoods, etc. The point is the students need to understand area covers or fills a certain amount of space with no gaps.

**Task Directions:**

The teacher will have two rectangles drawn and a simple closed curve with no straight sides on a sheet of paper. Make each one so that the three areas are not the same but with no area that is clearly largest or smallest. The students’ task is to first make a guess (estimate) about which is the smallest and the largest of the three shapes. After recording their guess, they should use a filler of their choice (color tiles, lima beans, or circular disks) to cover each. Each student should record in their math journal what they find. Following this, the teacher should have a whole group discussion about their findings.

The teacher will have the students reflect on the task they just completed. Since each drawing was drawn roughly close in size, write in your journal a response to the following formative assessment questions.

**FORMATIVE ASSESSMENT QUESTIONS**

• How did you determine the area of the shapes?
• Do you think that the area would be the same if you used different filler?
• How does the area change with the size of the filler?

**DIFFERENTIATION**

**Extension**

• The students can think of other fillers and estimate how many it would take to fill the three spaces and justify their estimate in relation to the original fillers that were used.

**Intervention**

• Using this task as a direct instruction strategy in small groups will provide support for students who struggle with these concepts and will enable them to develop the ability to describe their thinking.
CONSTRUCTING TASK: THE SAME BUT DIFFERENT

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

MCC3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

STANDARDS FOR MATHEMATICAL PRACTICE
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE AND MISCONCEPTIONS

As the teacher, your objective in the beginning is to develop the idea that area is a measure of covering. Do not introduce formulas. Simply have the students fill the shapes and count the units. Be sure to include estimation before measuring (this is significantly more difficult than for length), use approximate language, and relate precision to the size of the units in the same manner as with length (Van de Walle, page 237).

Area and perimeter (the distance around a region) are continually a source of confusion for students. Perhaps it is because both involve measuring length or because students are taught formulas for both concepts and tend to get formulas confused. Whatever the reason, expect that students even in the fifth and sixth grades will confuse these two ideas.

ESSENTIAL QUESTIONS

- What is area?
- Why are square units commonly associated with finding area?
- How does knowing the area of a square or rectangle relate to knowing different multiplication facts?
- Can one area measurement of a rectangle produce different dimension (factor) measurements?
• Also, can the same area measurement produce different size rectangles? (Ex. 24 sq.units can produce a rectangle that is a 3 X 8, 4 X 6, 1 X 24, 2 X 12)
• Do different dimensions with the same area cover the same amount of space? (Ex. Is a 3 X 8 the same area as a 1 X 24?)
• After covering each plane figure, what types of math could be used to determine how many squares were needed for covering?

MATERIALS

• Post-it notes
• one-inch tiles (color tiles)
• math journal/learning logs

GROUPING

Partner

TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION

In this task, students will create different area models for a given product.

Comments

The teacher will begin this lesson with a review of what is area. The teacher will then go back and reiterate how the classroom and our homes cover a certain amount of area. The students will be asked to look around the classroom. She will ask the students what 2 dimensional geometric plane figures do they see represented on the ceiling, walls, and floor. They should respond squares and rectangles. Allow time for the students to discuss other places that they’ve seen these two dimensional plane figures represented. The teacher will explain that area is measured in square units. Square units are the standard measurement for area. The class should have a discussion about why do they think these two figures are used most often and not triangles, trapezoids, or circles when building.

Task:

Tell the students that they will be placed into groups of two, and give each group a container of tiles or post-its. A number will be determined by the teacher and the students are to arrange the number of tiles or post-its into the shape of a rectangle. They can do it in whatever manner they chose. The only requirement is there can be NO GAPS or OVERLAPPING! The sides of each tile or post-it must TOUCH with NO SPACES! The teacher will need to call out numbers with more than two factors so there can be different representations. Ex. 12 can create a 2 X 6, 1 X12, 3 X 4. Once completed, tell each group to walk around the room and observe each other’s rectangles. Tell students to focus on another one in particular that is not like the one they designed. Following this, they will write a reflection in their math journal about how theirs is similar yet different to another student’s. The students should then have a discussion about their math findings. The teacher should offer more opportunities to explore by calling out more numbers and have the students walk and compare and contrast their area design with their peers. During this time, the teacher needs to be sure to reinforce that the rectangle is covering a
certain amount of space using square units. He/She can even relate it back to their homes and tiles on the bathroom walls, kitchen counters, ceiling tiles, etc. The connection must be made so that the students can see relevance. This will begin to set students up for area’s relationship to multiplication. Meaning, one number (product) can have different dimensions (factors) but the value is still the same.

The teacher will have the students think back to the tangram lesson and write a response to the following question:

- What is the relationship between the tangram activity and the activity just completed?

**FORMATIVE ASSESSMENT QUESTION**

- What did you notice about the area of your figure and your neighbor’s?
- How was your figure the same as your neighbor’s?
- How is your figure related to the tiles on the floor or ceiling?

**DIFFERENTIATION**

**Extension**

- As a set-up for the next day’s lesson, the teacher can have the students write in their journals ideas or strategies that can be used to find the area of a rectangle without having to count the number of tiles one-by-one. Some of the students could have figured out that they are skip-counting in two directions.

**Intervention**

- Using this task as a direct instruction strategy in small groups will provide support for students who struggle with these concepts and will enable them to develop the ability to describe their thinking.
CONSTRUCTING TASK: COUNT ME IN!

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

   b. A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units.

MCC.3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

MCC.3.MD.7. Relate area to the operations of multiplication and addition.
   a. Find the area of whole number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

The students should understand that a plane figure is a two-dimensional figure that consists of length and width with no height and covers a certain amount of space. In other words, it’s FLAT!

The students should understand what it means to skip-count and how skip-counting is repeated addition of the same number. Students should also be familiar with the term sum.

ESSENTIAL QUESTIONS

- What is area?
- Why are square units commonly associated with finding area?
- How does knowing the dimensions of a rectangle relate to area?
- How does knowing the dimensions of a rectangle relate to multiplication?
How does knowing the area of a square or rectangle relate to knowing different multiplication facts?

What makes an area model a good representation for multiplication?

By using an area model to learn multiplication, how many number patterns of multiplication are displayed?

How are multiplication and addition different? How are they the same?

What is the relationship between dimensions and factors?

What is the connection between area models and skip counting?

How is the commutative property of multiplication evident in an area model?

MATERIALS

- sidewalk chalk
- linking cubes or color tiles
- Post-its
- dry erase boards and markers
- math journal/learning log

GROUPING

Partner

TASK DESCRIPTION, DEVELOPMENT, & DISCUSSION

In this task, students will create area models and label them with appropriate dimensions.

Part I

The teacher can begin with a review of area. If the extension activity from the previous lesson was completed, which required the students to think of strategies to determine how much area was being covered WITHOUT counting 1, 2, 3; the teacher can have those students discuss their findings. Otherwise, students can use color tiles or even linking cubes and create a rectangle that is a certain area. Let’s say you begin with 24 square units. The teacher can walk around and have different students draw the different ways they represented 24 square units. Possible responses are a 1 X 24, 2 X 12, 3 x 8 and a 6 X 4.

The teacher then asks the following questions for depth and to lead to additions relationship to multiplication:

- While they look different, are they covering the same area? (This relates back to the Cover Me scaffolding task.)
- Let’s pretend the different area models you created for 24 were on the board. Excluding counting each square one at a time, what math strategies could you use to find out quickly the area being covered? (This question should lead the students to saying that they could have skip counted or some may respond that they would multiply.)
The teacher will then expound on area’s relationship to skip-counting/addition and multiplication. S/he will model and point out the following key concepts using an area model similar to the following, after allowing the students to watch the following video on Brainpop Jr. If your school does not subscribe to Brainpop Jr., a free trial is offered. Follow the link! http://www.brainpopjr.com/math/measurement/area/grownups.weml

![Area Model Diagram]

**Big Ideas Teacher Should Convey To Students During Instruction:**
- Area is related to addition in that the square units can be counted two ways: based on columns or length along with width or rows. This can be done using skip counting which is a form of addition.
- However, the fastest and easiest way is to do it by simply multiplying. Multiplying is repeated addition. Simply count across the top. In this case, it’s three. Then, count on the side to determine how long. This is 4. Explain to students that you have four rows with three in each, when turned you can display 3 rows of 4. This is multiplication or fast adding! (This will relate to **commutative property of multiplication**.)

**Part II**
The students will complete the “Count Me In” task recording form. Have the students share the different rectangles created and how they are similar/different.

**FORMATIVE ASSESSMENT QUESTIONS**
- What is the connection between skip-counting/addition and multiplication?
- Which one is better to use and why?
- Can the same areas look different? Why or why not?
- What is the commutative property of multiplication and how does it relate to area?
- What is the relationship between a product and a sum?
- Can an area measurement have the same area but different factors? How does that relate back to the tangram task?
- What would happen if we took some of the post-its or blocks away? Would we still have an accurate area measurement of the plane figure? Explain.
DIFFERENTIATION

Extension

- The teacher could take the students outside and break them into groups of two and give them 10 index cards with different multiplication problems. Using sidewalk chalk, she will have them draw an area model representing the multiplication fact and write the problem with the area measurement or product. She will stress the importance of putting square units after the product. **This is a fun activity that kids love and allows for lots of cooperative learning!**

Intervention

- Using this task as a direct instruction strategy in small groups will provide support for students who struggle with these concepts and will enable them to develop the ability to describe their thinking.
Create a rectangle with the area of 16 u². What is a multiplication sentence that could describe your rectangle?

Find a different way to show an area of 16u².

Draw a rectangle with the area of 36 u². What is a multiplication sentence that could describe your rectangle?

Find a different way to show an area of 36u².

Create a rectangle with the area of 30 u². What is a multiplication sentence that could describe your rectangle?

Find a different way to show an area of 30u².

Draw a rectangle with the area of 48 u². What is a multiplication sentence that could describe your rectangle?

Find a different way to show an area of 48u².
CONSTRUCTING TASK- “OOPS! I’M DECOMPOSEING!”

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.MD.7. Relate area to the operation of multiplication and addition.
  c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
  d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE AND MISCONCEPTIONS

Students should be able to fluently add and understand that addition is directly related to multiplication.

ESSENTIAL QUESTIONS

- What does it mean to decompose a number?
- How is the decomposition of a factor in an equation related to the distributive property of multiplication?

MATERIALS

- Centimeter grid paper (see task description for preparation instructions)
- Crayons
- Learning log/math journal
- http://mrsrenz.net/mathsites.htm#area-perim
- Base Ten Blocks (if possible alternate colors yellow/blue)

GROUPING

Partner
**TASK DESCRIPTION, DEVELOPMENT & DISCUSSION**

In this task, students will work through an array that is less than 10 x 10. The purpose is for the student to understand that numbers can be decomposed into “nice” numbers for multiplication and addition.

Each student/pair of students will be given cm grid paper (this will need to be prepared ahead of time) with rectangles drawn on them that are less than 10 x 10. Before the students attempt the area, discuss the dimensions of the rectangle. The example rectangle below is 11x8. The students will be given base ten blocks to determine the area. The decomposing of the numbers will be easier to see if the students have both the blue and yellow base ten blocks. For example blue rods and yellow units or something similar. It is important that the students are not instructed to use any particular base ten blocks. They may struggle but, will eventually figure out that it will be easier to tile with the rods than the units. When students have tiled the rectangle, ask them to respond to the following questions:

- What do you notice about the array?
- How did you determine the area?
- Were you able to see two or more rectangles in the larger one?
- Could you write a multiplication number sentence to identify the two rectangles?
- How does knowing the area of the two smaller rectangles help you to determine the number of the larger one?

Have students share their strategies with the class. Below is an example of a grid with possible solutions.

![Grid with possible solutions](image)

Students could see this as 10 x 8 and 1 x 8. Some may even see it as 8 tens is eighty, plus 8 ones. All solutions will give them an area of 88. In closing, you can talk about the relationship this has to the distributive property discussed in unit 3.

**FORMATIVE ASSESSMENT QUESTIONS**

- Were you able to see any additional arrays in the rectangle?
- How did you determine the area of the rectangle?
Was your strategy the most efficient?
What’s the relationship between the distributive equation and the original multiplication sentence?

DIFFERENTIATION

Extension
• For early finishers, the teacher could give them equations reflecting the distributive property of multiplication BUT have them figure out the original multiplication problem.

Intervention
• The teacher could do this lesson with small groups allowing them to use rectangles that are smaller in size (less than 100).
PRACTICE TASK: Multiplication with Base-Ten Blocks
Adapted from GPS frameworks

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.MD.7. Relate area to the operations of multiplication and addition.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a x b and a x c. Use area models to represent the distributive property in mathematical reasoning.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students need multiple experiences with base-ten blocks and how to represent ones, tens, and hundreds with them. Students should also understand how to trade pieces for equal values. For example, ten rods (of 10) can be traded for one flat (100).

Students need to have a good understanding of basic multiplication facts. They should also understand the various ways that multiplication number sentences can be written using an x, a dot, or parenthesis.

ESSENTIAL QUESTIONS

- How can base-ten blocks help us understand how to multiply a two-digit number?
- How does understanding the distributive property help us multiply large numbers?

MATERIALS

- Base-ten manipulatives for each student
- “Multiplication with Base-Ten Blocks” recording sheet

GROUPING

Individual/Partner Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION
In this task, students will model multiplication of 2-digit numbers using base-ten blocks to create partial products.

Comments
Students need to know more than one way to denote multiplication. The “x” may become confusing for some students when they begin using variables, so they should also recognize that a dot and parentheses are also symbols indicating multiplication.

Students need many experiences with arrays and base-ten blocks to be successful with this task.

Detailed examples follow below. Two colors are used to emphasize the placement of the base-ten blocks.

\[2 \times 13\] means there are two groups of 13. Using the base-ten blocks, ask students to build two rows of thirteen.

Have students make the row of 13 with one rod and three units joined together.

\[10 + 3 = 13\]

Repeat.

Place the two rows of thirteen in an array. The diagram below shows \[2 \times 13\] as two groups of 13 combined: two rods joined together, making two rows of ten, and six units joined together, forming two rows of three.

Students should see how to visually group the two rods to make twenty and the two rows of three units to make six, totaling 26.

In the next example, \[5(15)\] is five groups of fifteen. Have students build one row of fifteen with one rod and five units joined together.

Repeat four more times until they have five rows of 15. Join them together to form an array of five groups of fifteen.

Some students will quickly discover they can multiply the tens first, \[5 \times 10 = 50\], because the rods in the model are easy to see as groups of ten. Then they may see the units as an array, \[5 \times 5 = 25\]. Finally, they can add the two partial products, \[50 + 25\], to reach the total of 75.
As students practice while you model these examples, they often become quickly adept with this method. After sufficient practice with actual base 10 blocks, have them draw and label the arrays. Some will begin to do partial calculations in their heads and add them to get the totals much more quickly than they would with the traditional algorithm. This joining together of arrays clearly models the distributive property of multiplication.

Another way to think about the array is to describe it in terms of its dimensions of length and width.
For example, the same array can be shown as follows:

The 5 and 15 are shown as dimensions of the array, and can be described as “5 by 15.”
The area of the array is visibly shown as 50 + 25, or 75. This method of building arrays using dimensions reinforces the idea of the product shown as an area model and the dimensions as factors in the multiplication problem.

As students become more comfortable with this model, some will be able to move to using basic sketches to illustrate the model shown above. Rather than using grid paper or drawing each row, their sketches may evolve to look like the sketch shown below:

**Task Directions**
Students will follow the directions below from the “Multiplication with Base-Ten Blocks” recording sheet.

Model each expression with a drawing of base 10 blocks. Show how you use the model to find the product. Label the dimensions of each array. Write number sentences to help explain your drawings.

**Comments**
Students need the opportunity to work with manipulatives on their own or with a partner in order to develop the understanding of 2-digit multiplication. From the manipulatives, students will be able to move to pictorial representations of the blocks, then more abstract representations of the blocks (see the sketch above), and finally to abstract representation of multiplication using...
numbers. It is important to remember that this progression begins with concrete representations using manipulatives.

**FORMATIVE ASSESSMENT QUESTIONS**

- How did you know which pieces and how many to use for your array model?
- What partial products did you create?
- How does the arrangement of the base-ten pieces help you see partial products?
- What are the dimensions of your array?
- What product/area does your model represent?

**DIFFERENTIATION**

**Extension**
- Give students a base-ten block array or a drawing of an array and have them determine the product and its factors.
- Have students decide on a number, build it with base 10 blocks, and then trade seats with a neighbor to determine the factors and find the product.
- Have students use an array to write/solve division problems.

**Intervention**
- Begin with much smaller arrays, such as 2 x 3, 3 x 4, and 2 x 6. Have students describe the dimensions and area of each array. Then connect dimensions and area to the actual multiplication sentence.
- Use grid paper and allow students to place the base-ten blocks onto the grid paper first and then to count the grid squares as part of their calculations.
- If necessary, allow students to use a times table chart or other cueing device if full mastery of the basic multiplication facts has not yet been attained.
Multiplication with Base-Ten Blocks

Model each expression with a drawing of base 10 blocks. Show how you use the model to find the product. Label the dimensions of each array. Write number sentences to explain your drawings.

4 x 14

12 • 7

5(15)

(13)(6)
PRACTICE TASK: Array Challenge

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

MCC3.MD.7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

The students should know that area covers a certain amount of space. The students should know that the numbers that are multiplied to find a product are called factors.

It is important to understand that filling regions with units and counting does little to help students develop multiplicative formulas. Even when rectangles are filled with a grid of squares, students are more likely to count the squares than to relate the number of squares to the dimensions of the rectangles (Van de Walle, page 236-237).

ESSENTIAL QUESTIONS

- How does knowing the length and width of a rectangle relate to multiplication?
- Can the same area measurement produce different size rectangles? (Ex. 24 sq.units can produce a rectangle that is a 3 X 8, 4 X 6, 1 X24, 2 X 12)
- How does the length and width (factors) impact the area of the rectangle?
- Do different factors with the same area cover the same amount of space? (Ex. Is a 3 X8 the same area as a 1 X 24?)
MATERIALS

- “Shaded Array Cards” copied on card stock and cut out
- “Array Challenge” game directions and recording sheet

GROUPING

Partner/Small Group

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students work in small groups to play a game in which array cards are used to represent area models for multiplication facts. Students have opportunities to display their cards and respond with the multiplication fact(s) that apply to the array.

Comments

The Shaded Array Cards provide an excellent opportunity for students to make visual connections between multiplication facts and the corresponding area models. Students are able to relate the commutative property of multiplication to the model quickly because it represents a fact and its related fact. For example, the area model for $6 \times 7$ is the same as $7 \times 6$ with a different orientation. Also, familiarity with array models for multiplication facts builds number sense as students understand that a smaller array represents a smaller product of two facts.

\[
\begin{array}{c}
6 \text{ rows of 7 or } 6 \times 7 = 42 \\
\end{array}
\]

\[
\begin{array}{c}
7 \text{ rows of 6 or } 7 \times 6 = 42 \\
\end{array}
\]

Task Directions

Have students follow the directions below:

1. Place the Array Cards face down in a stack.
2. For each round, each player should draw one card from the stack and, using the commutative property, record both multiplication facts that apply to the card. (If the array is a square, there will be only one multiplication fact for the array.)

3. At the end of each round, the player with the largest product collects the cards from the other players.

4. Play continues until all cards have been played.

NOTE: The rules can be changed so that the player with the smallest product collects all the cards.

FORMATIVE ASSESSMENT QUESTIONS

- How can you use your Array Card to show the commutative property for multiplication?
- How does the size of the array change as the factors get larger? Smaller?
- How are the dimensions of the array and the number of shaded squares related?
- How does an array model show repeated addition?

DIFFERENTIATION

Extension
- Make additional Array Cards that model higher levels of multiplication facts.
- Play Double Challenge where students draw two cards at a time and add the products.
- Have students use the Array Cards to explain the division facts that are related to a given array and write the corresponding fact family for multiplication and division.

Intervention
- Make Array Cards with lower level multiplication facts, or with other math facts and concepts that students need to review.
- Use this game in small group instruction to informally assess a student’s level of multiplication fact mastery and to pinpoint specific areas to target instruction.
Array Challenge

Game Directions

Array Challenge is a game for 2 – 4 players.

Materials:
One deck of Array Challenge cards
Array Challenge recording sheet

Directions:

1. Place the Array Cards face down in a stack.
2. For each round, each player should draw one card from the stack and, using the commutative property, describe both multiplication facts that apply to the card. (If the array is a square, there will be only one multiplication fact for the array.)
3. At the end of each round, the player with the largest product collects the cards from the other players.
4. Play continues until all cards have been played.

NOTE: The rules can be changed so that the player with the smallest product collects all the cards.

Record the multiplication facts for your array cards in the table on the back of this sheet.
Example: If you drew a 6 x 7 array card, two number sentences can be written.

6 rows of 7
or
6 x 7 = 42

7 rows of 6
or
7 x 6 = 42
Array Challenge
Recording Sheet

Record the number sentences for each array card in the table below.

<table>
<thead>
<tr>
<th>Round</th>
<th>Number Sentence</th>
<th>Number Sentence</th>
<th>Highest Product?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>$6 \times 7 = 42$</td>
<td>$7 \times 6 = 42$</td>
<td>✓ or ✗</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>12</td>
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</tr>
</tbody>
</table>
Shaded Array Cards
CONSTRUCTING TASK: Skip-Counting Patterns

STANDARDS FOR MATHEMATICAl CONTENT

MCC3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Multiplication facts should be mastered by relating them to existing knowledge. It is essential for students to understand the commutative property. For example, 2 x 7 is related to the fact double seven. However, the same relationship applies to 7 x 2 that most think about as 2+2+2+2+2+2+2. Many of the facts are easier to master in one order, but should always be learned with its turn around. Van de Walle indentifies the following patterns; doubles, fives facts, zeros and ones, and nifty nines. These rules cover 75 of the 100 facts. (Teaching Student Centered Mathematics, Van de Walle, John, A., p.88-89)

This activity provides opportunities for the students to make sense of the many patterns in our base-ten system and how it is full of patterns. It helps with multiplication and division as well as providing time to search for patterns.

ESSENTIAL QUESTIONS

- How can multiplication products be displayed on a 1-100 chart?
- How can you describe various patterns, (i.e. with words, as a visual pattern on a 1-100 chart, or using mathematical notations)?

MATERIALS

- “Skip-Counting Patterns, Directions” student sheet
- “Skip-Counting Patterns, 1-100 Chart” student sheet (Students can cut apart the 1-100 charts to create a small booklet.)
- A large 1-100 chart that can be used for class discussion
- Highlighters, crayons, colored pencils, or markers for each student
GROUPING

Individual Task

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students create patterns on a 1-100 chart by skip counting with each number 2 through 10. Students do this by coloring in each number on which they land. Afterwards, students look for patterns on their charts and discuss how the patterns identified inform their understanding of our base-ten system.

Comments

One way to introduce this task is by asking students to identify patterns on the 1-100 chart. Examples of student observations may include:

- All of the numbers in the first column end in a 1 (also noticing the other columns and the observation that the last digits remain the same.)
- All of the numbers in the last column are the ones we say when we count by 10s.
- When you start in the top left corner and go down diagonally the one’s digit goes up by 1 and so does the tens digit.

During this task, students will highlight or mark all numbers they land on when skip-counting by 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, and 10s. Students will use a clean 1-100 chart for each number and start on the number by which they are counting (e.g. when counting by 2s, they will start on 2). Once they finish highlighting a 1-100 chart, students should discuss with a partner or partners any patterns they notice. Once students have completed skip-counting by the numbers 2-10, initiate a class discussion about the patterns students observed. Create a class list of student observations. As an example, student observations of patterns when counting by 2s on the 1-100 chart could include:

- Only the even numbers are highlighted.
- All of the shaded numbers are in the ‘even’ columns.

Encourage students to check their work as they go with other students, a calculator, or by referring to a teacher-created sample so that students don’t get frustrated. Obviously, one error on the 1-100 chart will result in all of the subsequent numbers being incorrect as well.

Task Directions

Students will follow directions below from the “Skip-Counting, Directions” student sheet.

You will be skip-counting by 2, 3, 4, 5, 6, 7, 8, 9, and 10.

1. Highlight or mark all numbers counting by 2s starting with 2 (i.e., skip counting by twos). Discuss with your partner(s) what you notice about the highlighted numbers.
2. Using a new hundred chart, highlight or mark all numbers counting by 3s starting with 3. Discuss with your partner(s) what you notice about the highlighted numbers.
3. Continue with a new hundred chart for each number 4 through 10, highlighting the numbers you land on as you skip count by each number. After completing
each chart, discuss with your partner(s) what you notice about the highlighted numbers.

4. Be prepared to share your observations about patterns on your 1-100 charts with the class.

FORMATIVE ASSESSMENT QUESTIONS

- How do you know you skip-counted correctly?
- What do you notice about the numbers that are highlighted?
- How can you describe the geometric pattern that is formed with the highlighted numbers?
- After the students have skip-counted by the first few multiples (2’s, 3’s, 4’s): When you skip-count by 5 or 6 will you have the same, more, or less, numbers highlighted than when you skip counted by 2?

DIFFERENTIATION

Extension
- Ask students to compare the two 1-100 charts they created. For example, compare the 2’s and 4’s chart and ask the students to describe what they notice about these two charts and more importantly why this is happening. One way students could organize their thinking is by recording the highlighted numbers in a Venn Diagram and then writing about what they notice and why their observations make sense based on our base-ten number system.
- Ask students to make predictions before they compare the various charts, such as, “Will the 6’s and 9’s have anything highlighted in common? Why or why not?” Or “How do you know?”

Intervention
- Encourage students to use a calculator (or another tool) to determine the highlighted numbers. Being off by one number can be very frustrating and the main objective of this task is not to generate the numbers when skip-counting but to analyze the numbers found.
## Skip-Counting Patterns

### Directions

You will be skip-counting by 2, 3, 4, 5, 6, 7, 8, 9, and 10.

1. Highlight or mark all numbers counting by 2s starting with 2 (i.e., skip counting by twos). Discuss with your partner(s) what you notice about the highlighted numbers.

2. Using a new hundred chart, highlight or mark all numbers counting by 3s starting with 3. Discuss with your partner(s) what you notice about the highlighted numbers.

3. Continue with a new hundred chart for each number 4 through 10, highlighting the numbers you land on as you skip count by each number. After completing each chart, discuss with your partner(s) what you notice about the highlighted numbers.

4. Be prepared to share your observations about patterns on your 1-100 charts with the class.

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Skip-Counting Patterns

1 – 100 Charts

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PRACTICE TASK: TAKE THE EASY WAY OUT!

Portions of this lesson were adapted by Elizabeth Wistrom and Donna Cosmato.

STANDARDS FOR MATHEMATICAL CONTENT:

MCC.3.OA.9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE AND MISCONCEPTIONS

Learning multiplication facts does not seem as daunting when you tell your students they will only have to memorize a total of 10 facts! How is this possible, they might ask? The answer is multiplication patterns.

From very early on, children have been taught to identify patterns - in reading, in spelling, in art, in music and of course, in mathematics. The Multiplication Table is no exception. It is made almost entirely of repeating patterns. Once these patterns are identified and understood, it can be noted that there are only 10 remaining multiplication facts that do not fit a specific pattern. For these problems, the only available learning tool is memorization. Still...the thought of having to memorize 10 problems is much less overwhelming than the thought of memorizing an entire table!

Misconceptions

There are many misconceptions about how students should learn their multiplication facts. Most think that multiplication facts should be taught in numerical order starting with 0 and 1, then facts less than 5, and from there on out in order, facts 6 through 12. They also believe that multiplication should be taught in isolation with no connection to addition. A third misconception is that multiplication should be taught separately from division. (Wallace & Gurganus, 2005).
Many times these misconceptions are based on a teacher’s or parent’s personal learning experience. Fortunately, we now know from the research of Van de Walle, Fosnot et. al., Heibert, and others that there are more effective strategies for teaching and learning the multiplication facts. Several of these strategies are outlined in Teaching Student Centered Mathematics, by John A. Van de Walle.

**ESSENTIAL QUESTIONS**

- What is a pattern?
- How are patterns related to multiplication?
- How can an addition table help you explain the Commutative Property of Multiplication?

**MATERIALS**

- Construction paper and writing paper
- Hundreds charts
- *Amanda Bean's Amazing Dream (A mathematical story) - Cindy Neuschwander*
- *The Mathemagician's Apprentice - Brian Boyd*
- *The Best of Times - Greg Tang*
- *Grapes of Math - Greg Tang*
- Math journal/learning log
- Printable Task Games

**GROUPING**

Individual/Partner

**TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSION**

In this task, students will explore patterns in multiplication. They will also create a book of multiplication rules to learn relationships that exist within the multiplication family and apply the concepts to a hundreds chart.

By showing your students how to use multiplication patterns, you will demonstrate that it is really only necessary to memorize 10 multiplication facts. The lesson could open with one of the above multiplication books. Each book is about patterns in multiplication. However, the ability levels are different. Select the book that best fits the level of your class. Following the reading of the book, engage the students in the following task:

Tell students they are going to create their own multiplication book. Provide each student with two pieces of construction paper and eight sheets of manuscript paper. This will serve as the basis for your book. The title of the book will be *My Multiplication Book*. The first page of the book should define the word “multiplication”:
What is multiplication?
After eliciting responses from your students, you should (together) construct a definition that reads something like this:

- Multiplication is a way to add groups of equal size. $3 \times 4 = 12$ (How many groups x How many in each group = How many all together)

Have your students write the agreed upon definition on the first page of their multiplication book.

The next step is to have students explain the Commutative Property of Multiplication, and determine a definition:

When two numbers are multiplied together, the product is the same regardless of the order of the factors. For example $3 \times 2 = 2 \times 3$.

By understanding this principle, students see that many problems can be overlooked because they are actually duplicates. Have them write the agreed upon definition on the second page of their book.

The following pages of the book will cover the fact families that show definitive patterns, the associated rules, the numerical sentences that make up the fact family and a pictorial representation of the Commutative Property of Multiplication. Below you will find examples of each of these pages.

My Multiplication Book
Using a Multiplication Table as a Visual Aid
An effective way to manage the lessons is to introduce one Rule/Family each day. Using a large multiplication table can be a terrific visual aid. As you learn each fact family, cross them off on the multiplication table. That way, your students can actually see the progress they are making in learning the multiplication facts:

Multiplication Table after the 1’s Family is studied

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Only 10 Problems Left!
The last page of your Multiplication Book will include the 10 multiplication problems that are not covered by the fact family patterns studied:

3x3, 3x6, 3x7, 3x8, 6x6, 6x7, 6x8, 7x7, 7x8 & 8x8
(Remember...due to the Commutative Property of Multiplication, there are only 10 problems instead of 20.)

This is how the multiplication table you are using for demonstration will look when you have completed teaching multiplication facts. (The circled problems are the 10 that must be learned through an alternative strategy)

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### Task Directions:
After teaching the 2’s and 4’s pattern, have the students do the following task:

**2’s Rule!**- Have the students work in groups of two and supply them with the attached sheet called 2’s Rule! This is a game. The students will cut out the cards and place them face down in a pile. They will take turns flipping over a card 1 at a time. However, the first person to give the correct answer wins the card. The person with the most cards wins the game. Since the 2’s rule is double the number and add, some of the cards will simply have an even number. The students must then tell what number was doubled and say the 2’s multiplication sentence that correlates with it. Van de Walle states that division should be taught in connection with multiplication.

**4 Score 4 Sure!**- This task can be completed as a 4 man game, which will be intense fun! Or, it can be a cut and paste activity with a partner. Break students into teams of 4. Give them the attached reproducible page and have them cut out the numbers. Once completed, revisit the rule of 4. Then, have the groups move to separate locations around the room. Rules: The teacher will say a number. Let’s say it’s 3. She can say, “I need to see this number’s double, and then its double.” Thus, working cooperatively, the students must send up two people from the team holding up the number 6 and 12. For an extra point, the teacher can have them give the 4’s multiplication sentence that corresponds with 3 which would be 4 times 3.

To advance the game, the teacher could make them think backward. She could only give the product and they would have to send up two people with half the product and then half of that. This would once again, as Van de Walle states, show connections to division while still reinforcing multiplication and teaching pattern relationships. Of course, the same procedure for the extra point would apply. The team with the most points wins. **(Suggestion:** The numbers for the games can be written on large index cards or construction paper so they are visible to all learners during the game.)
FORMATIVE ASSESSMENT QUESTIONS

- How does learning patterns aid mastery in multiplication?
- How does the commutative property of multiplication aid mastery of multiplication facts as well?

DIFFERENTIATION

Extension
- The lessons could be extended by creating similar tasks or games using different factors and products.

Intervention
- Having the students work in small groups will provide support for students who struggle with this concept and will enable them to develop the ability to describe their thinking.
Math Book Synopsis

Amanda Bean's Amazing Dream (A mathematical story) - Cindy Neuschwander

Amanda Bean happily counts "anything and everything" by ones, twos, fives, and tens. Although her teacher tells her that learning multiplication is important, Amanda remains unconvinced until a strange dream presents her with arithmetic challenges that overwhelm her counting skills. She awakens to learn to multiply "anything and everything." Recommended for 6-8 yrs but another fun introduction.

The Mathemagician's Apprentice - Brian Boyd

Oz, the mathematician's apprentice, needs help with his final test. Teaching times tables whilst you help Oz. Packed on each page with an activity to do, the book also includes a CD. Princess really enjoyed working with this, and needed Michelangelo's help.
The Best of Times - Greg Tang

Greg Tang uses rhymes and commonsense tricks to walk through the multiplication tables from zero to 10. For example, if you know how to multiply by two ("Two is very fast and fun, quickly double and you're done. What's that you say, be more precise? Okay then, just add it twice!"), then fours ("... please just always double twice!") and eights. This book does not promote the memorization of multiplication facts, but teaches the reader to problem solve and use different approaches and strategies. Does move fast but we found it excellent! 5 Star.

Grapes of Math - Greg Tang

Tang shows readers creative ways to use patterns and combinations of numbers to solve math puzzles quickly and effectively. Rather than laboriously counting 24 mushroom slices on a pizza, Tang suggests: "Let me give you some advice, / Just do half and count twice." And in adding the number of dots on a fan: "Instead of seeing groups of threes, / Count by fives and it's a breeze!"
Each riddle offers a clue, the "Answers" section at the back of the book, offers an explanation of each problem and shows how to group objects together and look for patterns. Recommended for 8-10 years. An easier introduction than 'The Best of Times.' Excellent!
### 2's Rule!

<p>| | | |</p>
<table>
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<tbody>
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<td>2 X 8</td>
<td>10</td>
</tr>
<tr>
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<td>2 X 9</td>
<td>12</td>
</tr>
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<td>2 X 10</td>
<td>14</td>
</tr>
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<td>18</td>
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<td>6</td>
<td></td>
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<td>2 X 7</td>
<td>8</td>
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</table>
### 4's The Score!

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<tr>
<td>6</td>
<td>24</td>
<td>18</td>
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</tbody>
</table>
CONSTRUCTING TASK: READ ALL ABOUT IT

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

MCC.3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

MCC.3.MD.7 Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

“Problem solving focuses students’ attention on ideas and sense making. It allows students to make sense of mathematics, while providing an ongoing assessment for the teacher. It is important that students explain and defend their solutions. The information gained from this will allow teachers to clear up misconceptions and plan for future instruction. A good problem solving task allows for multiple paths to a solution.” (Van de Walle, Teaching Student-Centered Mathematics, p. 15)
ESSENTIAL QUESTIONS

- How can we use patterns to solve problems?
- How do estimation, multiplication, and division help us solve problems in everyday life?
- How do rectangle dimensions impact the area of the rectangle?

MATERIALS

- “Read All About it” task sheet
- 1 inch color tiles
- 8 ½ x 11 inch paper

GROUPING

Small group/partner

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

The teacher will present the students with the following problem solving task:

The 3rd grade class at Georgia Elementary School wanted to go on a field trip to a soda factory. The trip will cost $100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. How many pages long will the newspaper be if they used paper that was 8 ½ x 11 inches? Will there be enough room for additional graphics on the pages once the articles have been written? How did you determine this?

Students should be allowed to use 1 inch color tiles as well as sheets of paper to complete the task. They should show their solution using pictures, numbers and words.

FORMATIVE ASSESSMENT QUESTIONS

- How did you determine the number of pages needed?
- Is there another way you could have solved this?
- Did you find a pattern when you were solving this?
- How does your knowledge of area help you solve this problem?

DIFFERENTIATION

Extension
- The students could determine the cost of producing the paper, and how many copies should be sold and at what price, in order to reach their goal.

Intervention
- Decrease the number of students that are writing articles.
- Use this task in a guided small group.
READ ALL ABOUT IT

The 3rd grade class at Georgia Elementary School wanted to go on a field trip to a soda factory. The trip will cost $100. The students decided to write a class newspaper and sell it to the kids at their school. Each of the 20 students will be given a 16 inch square for his/her article in the newspaper. How many pages will the newspaper be if they used paper that was 8 1/2 x 11 inches? Will there be enough room for additional graphics on the pages once the articles have been written? How did you determine this? Use pictures, numbers, and words to show your solution and your mathematical thinking.
CONSTRUCTING TASK: Armadillo Stories

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.OA.8 Solve two-step problems involving the four operations, and identify and explain patterns in arithmetic. Solve two - step word problems, using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students need a good understanding of the components of a number sentence, the use of a symbol to represent what is being found, and how to translate between words and mathematical symbols.

ESSENTIAL QUESTIONS

- What math is involved in the study of Georgia animals?
- How can multiplication help us repeatedly add larger numbers?

MATERIALS

- “Armadillo Stories” recording sheet
- Manipulatives, if needed
- Research resources such as informational text and/or the internet

GROUPING

Individual/Partner Task
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

In this task, students will write and solve word problems and their accompanying number sentences using a given data set. They will also include symbol(s) in their number sentences.

Comments
Animals are usually highly motivating subjects for third graders to study. Be sure they note how science and mathematics are connected as they study Georgia animals and habitats throughout the school year.

As students solve their multiplication story problems, have them verbalize what each number in their number sentence represents. In the example on the recording sheet, the number sentence is $15 \times 10 = 150$ inches. Be sure students can explain that the 15 represents the length of the armadillo’s tail in inches and the 10 represents the number of tails.

Task Directions
Students will follow the directions below from the “Armadillo Stories” recording sheet.

Armadillos are animals native to Georgia, and are they ever strange! Think about these armadillo facts:

- Armadillos live an average of 12 to 15 years.
- An armadillo can be as long as 59 inches.
- An armadillo’s tail is about 15 inches long.
- An armadillo can jump nearly 5 feet straight into the air.
- The largest armadillos weigh 120 pounds.
- An armadillo mother has 4 identical armadillo babies every time she gives birth.


These armadillo facts can be used to write multiplication stories.

**Example:**

*If the tails of 10 average armadillos were placed end to end, how long would they be?*

*One armadillo tail is 15 inches long.*

*There are 10 armadillos.*

*My number sentence is: $15 \times 10 = \square$ inches.*

*The tails of ten armadillos put together would equal 150 inches.*

**Example:**

*Four armadillos weigh 480 pounds. How much does one armadillo weigh?*

*My number sentence is: $4 \times \square = 480$ pounds*

*$4 \times 120 = 480$ pounds*

*Each armadillo weighs 120 pounds.*
Write and solve three more multiplication stories about armadillos or another interesting Georgian animal.

**FORMATIVE ASSESSMENT QUESTIONS**

- What data did you use for your word problem?
- How did you decide what to include in your number sentences?
- Is there more than one correct way to write your number sentence? How do you know?
- How did you use a symbol in your number sentence? What does it represent?
- What does each part of the multiplication sentence represent in your story?
- How does multiplication help us represent ideas about the sizes of armadillos?

**DIFFERENTIATION**

**Extension**

- Have students discuss and make a list of the ways that measurements are used in science. Have them construct a chart to show both the English and the metric (when applicable) measures of length and width, time, speed, and temperature.
- Encourage students to experiment with writing two step word problems.

**Intervention**

- Have students model their word problems (using different numbers) on the sample problem given or a problem that the teacher demonstrates.
- For kinesthetic learners, allow them to use math magnets or other manipulatives to set up their math sentences on a surface that is easily manipulated prior to recording the number sentence.
Armadillo Stories

Armadillos are native Georgia animals and are they ever strange!
Think about these armadillo facts:

- Armadillos live an average of 12 to 15 years.
- An armadillo can be as long as 59 inches.
- An armadillo’s tail is about 15 inches long.
- An armadillo can jump nearly 5 feet straight into the air.
- The largest armadillos weigh 120 pounds.
- An armadillo mother has 4 identical armadillo babies every time she gives birth.

These armadillo facts can be used to write multiplication stories.

Example:
If the tails of 4 average armadillos were placed end to end, how long would they be?

One armadillo tail is 15 inches long.
There are 4 armadillos.

My number sentence is: $15 \times 4 = \square$ inches.
The tails of ten armadillos put together would equal 60 inches.

Example:
Four armadillos weigh 360 pounds. How much does one armadillo weigh?

My number sentence is: $4 \times \square = 360$ pounds
$4 \times 90 = 360$ pounds
Each armadillo weighs 90 pounds.

Write and solve three more multiplication stories about armadillos or another interesting Georgian animal.
CONSTRUCTING TASK: SUBJECT TO INTERPRETATION!

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

How Should We Teach Data Analysis? (Van de Walle, 2007)

• Don't rely on textbook questions- students are not interested in questions that have no relevance to their own lives. Allow opportunities for students to generate questions, decide on the appropriate data needed to answer the question, and how to analyze the data.

• In the primary grades, students like to learn about themselves and their class. Questions around favorites, numbers, and measures are useful for the primary grades. Each student can contribute one piece of data (favorite color, number of siblings, foot length).

• In the upper elementary grades, students can answer questions outside of the classroom. At these grade levels, data collection and analysis can be used in science and social studies (collecting leaf samples, U.S. Census).

• Students must learn how to categorize or classify things in order to organize the data.

• Classification is done by finding objects’ attributes that allow them to be sorted (color, height, gender, etc.).

• Once data is organized, it can be recorded in a graphical representation such as a pie chart, a picture graph, a bar graph, a line plot, or a histogram.

• Students should construct their own graphs so that they are connected to the data and they learn how a graph conveys information about the data.
• These graphs provide a visual representation that allows students to look at information about the data as the “big picture” and see how the data is spread out, instead of just a collection of numbers.

• It is possible for one set of data to be used to answer multiple questions about the population depending on the depth of the data collection.

• Understanding of data is impacted by choice of graphical representation.

• The shape of the data allows the student to see how the data is spread out or grouped, what characteristics of the data is seen with the graph, and what it says about the data's population.

**ESSENTIAL QUESTION**

• What is the relationship between a pictograph and problem solving?
• What’s the relationship between the picture’s value and patterns found in multiplication?

**MATERIALS**

• Math journals/learning logs
• clipboards
• Chart paper
• markers

**GROUPING**

Partner

**TASK DESCRIPTION, DEVELOPMENT, AND DISCUSSIONS**

In this task, students will pose a question, collect and organize data, and analyze and interpret data.

Students should have opportunities reading and solving problems using scaled graphs before being asked to draw one. While exploring data concepts, students should Pose a question, Collect data, Analyze data, and Interpret data (PCAI).

The teacher can begin the lesson by asking the students the following questions: What if I needed to order math team t-shirts for 40 students? What are some possible strategies I could use to collect the information and report about which of four t-shirt styles they preferred?

**Pictographs:** Scaled pictographs include symbols that represent multiple units. The teacher will explain the purpose of posing, collecting, and organizing the data first. Using chart paper, the teacher should model the creation of a pictograph for the t-shirts. The teacher should emphasize the importance of graphs having a title, categories, category label, key, and data.
## T-Shirt Colors

<table>
<thead>
<tr>
<th>T-Shirt Colors</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Striped T-shirts</td>
<td></td>
</tr>
<tr>
<td>Blue T-shirts</td>
<td><img src="Blue_T-shirts.jpg" alt="Image" /></td>
</tr>
<tr>
<td>Yellow T-shirts</td>
<td><img src="Yellow_T-shirts.jpg" alt="Image" /></td>
</tr>
<tr>
<td>White T-shirts</td>
<td><img src="White_T-shirts.jpg" alt="Image" /></td>
</tr>
</tbody>
</table>

= 5 T-Shirts

The teacher could then model analyzing and interpreting the data through questions:

- Which t-shirt did they like most?
- Which did they like least?
- How many more people liked white than yellow?
- Which t-shirt was liked more than striped but less than blue?
- What interval was used for each t-shirt?
- What are the benefits of using a pictograph?
- Is there a relationship to multiplication? If so, what is it?
- Can the picture values vary or change?
- How does this relate to multiplication and patterns?
- How does the pictograph help solve problems?

**Note to teacher:** Use the following pictograph to develop student understanding of scaled graphs. Students need opportunities reading and solving problems using scaled graphs before being asked to draw one.
Girl Scout Cookie Pictograph
Four Girl Scouts sold cookies for one month. The list below shows how many boxes were sold by each Girl Scout.

Jamiya - 60 boxes
Lauren - 40 boxes
Zoey - 25 boxes
Macy - 15 boxes

Use the information from the list to complete the pictograph below and answer the questions.

<table>
<thead>
<tr>
<th>NAME</th>
<th>Cookie Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Macy</td>
<td></td>
</tr>
<tr>
<td>Lauren</td>
<td></td>
</tr>
<tr>
<td>Jamiya</td>
<td></td>
</tr>
<tr>
<td>Zoey</td>
<td></td>
</tr>
</tbody>
</table>

= 5 boxes

1. How many boxes of cookies did the girls sell in all? _______________________

2. How many more boxes of cookies did Jamiya sell than Macy?___________________

3. Which two girls sold a total of 65 boxes of cookies?

4. Half of the cookies sold by Lauren were Thin Mints. How many boxes of Thin Mints did Grace sell?__________________________

5. How many more cookies did Zoey and Macy need to sell in order to equal Jamiya?________
TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

The teacher will instruct the students to generate a question to use as a survey question. Using a clipboard and the attached sheet, they will ask the question, record/collection the data, and create a pictograph so they can analyze and interpret the data. The teacher should tell them that each picture symbol was worth 2 this time and explain the need of cutting the picture in half if it is an odd number.

The students will create a pictograph along with questions, switch with another group and allow them to answer the questions and vice-versa, and conclude by exchanging back and grading each other’s work being sure to discuss mistakes and what should have been included or excluded.

FORMATIVE ASSESSMENT QUESTIONS

- What is the relationship between a pictograph and problem solving?
- What’s the relationship between the picture’s value and patterns found in multiplication?

DIFFERENTIATION

**Extension**

- The teacher could increase the value of the scale intervals to numbers beyond 10 to challenge students who are fluent with their multiplication facts. OR, she can have the students import their data into an online pictograph generator using the following website: http://illuminations.nctm.org/ActivityDetail.aspx?ID=204
- The same concept used to teach pictographs can be applied to teach simple bar graphs.

**Intervention**

- Having the students work in small groups will provide support for students who struggle with this concept and will enable them to develop the ability to describe their thinking.
**Pictograph Data Collection Sheet**

TOTAL NUMBER OF PEOPLE SURVEYED:____________________

SURVEY QUESTION:______________________________________________________

DIRECTIONS: Interview the total number of people in your class and use a tally mark to represent their response. Use the attached box as needed to record the data and create the bar graph.

---

**Title of Bar Graph**

<table>
<thead>
<tr>
<th>Survey Responses</th>
<th>Tally Marks</th>
<th>Total</th>
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</tbody>
</table>

**Hair Color**

<table>
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<th>Hair Color</th>
<th>Tally Marks</th>
<th>Total</th>
</tr>
</thead>
<tbody>
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<tr>
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</tr>
<tr>
<td>Blond</td>
<td></td>
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<tr>
<td>Red</td>
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</table>

**Eye Color**

<table>
<thead>
<tr>
<th>Eye Color</th>
<th>Tally Marks</th>
<th>Total</th>
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<tbody>
<tr>
<td>Brown</td>
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<tr>
<td>Blue</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Green</td>
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</tbody>
</table>
CONSTRUCTING TASK- MEASURE AND PLOT!

STANDARDS FOR MATHEMATICAL CONTENT

MCC.3.MD.4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE & MISCONCEPTIONS

Bar graphs and picture graphs are useful for illustrating categorical data. A line plot is used for counts of things along a numerical scale. It is essentially a number line with an X placed above the corresponding data. The advantage to a line plot graphs is that all data is displayed.

ESSENTIAL QUESTIONS

How is a line plot similar to a bar graph?
How are they very different?

MATERIALS

- Rulers, tape measures, yard sticks
- math journals/learning logs
- http://www.youtube.com/watch?v=G6eTMRXHhmE

GROUPING

partner

TASK DESCRIPTION, DEVELOPMENT & DISCUSSION

In this task, students will measure in to nearest whole inch and create a class line plot graph.
Comments
The teacher will review the purpose of a bar graph through an open-ended discussion. She will explain that there are many different types of graphs and the one they will be studying today was called a line plot graph.

The teacher would then show them the video below which will explain the purpose of a line plot and how to create the graph. http://www.youtube.com/watch?v=G6eTMRXHhmE

Following the video, the teacher will display a Venn diagram and have the students compare and contrast a bar graph and a line plot graph. The teacher would then display the following numbers on the board and have them work as a whole group to create a line plot. She would be sure to give it a title to reinforce the connection between bar graphs, tell them the numbers they were using were grades from a spelling test from last year’s class which means that is the data, and go on to model creating the plot.

(85, 90, 75, 100, 100, 80, 60, 100, 90, 90, 100, 65, 60, 85, 90, 75, 85, 65, 100, 90)

Task
The teacher will explain (distribute the recording sheet) the following task to the students:

The custodians will be adjusting the height of the swings on the playground. They have decided to take data from the third grade classes. The custodians will need to know the sitting height of a majority of the third grade students. They will use this data to adjust the swings.

Students are to work in pairs to determine their sitting height to the nearest inch. Students should record their height on their task recording sheet.

When the measuring is complete, the students should work together to create a class line plot graph. There should be discussion around the first and last number in the line plot and how this will be determined.

FORMATIVE ASSESSMENT QUESTIONS

- What makes a bar graph different from a line plot?
- Give examples and explain when would be the most appropriate time to use each and why.

DIFFERENTIATION

Intervention-
- Students may struggle with measuring. Provide assistance. Using a tape measure may be easier than a ruler or yard stick. Also, notice where students begin their measurement. Are they starting at one or zero?

Extension-
- Students can collect data from the other third grade classrooms for a grade level line plot.
Measure and Plot

The custodians will be adjusting the height of the swings on the playground. They have decided to take data from the third grade classes. The custodians will need to know the sitting height of a majority of the third grade students. They will use this data to adjust the swings.

My sitting height____________________

What are three things the data tells me about the sitting height of my classmates?

Do you think that it was wise to use the third grade to determine the swing height? Explain your answer.
PRACTICE TASK: HOOKED ON SOLUTIONS!

STANDARDS FOR MATHEMATICAL CONTENT

MCC3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity.

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

BACKGROUND KNOWLEDGE

Students should understand math concepts addition and subtraction and multiplications relationship to addition.

ESSENTIAL QUESTIONS

- How can multiple math operations be used to solve real world problems?
- How can we use patterns to solve problems?
- Why is it important to understand that more than one math operation may be needed to solve a problem?

MATERIALS

Unifix cubes, or any counting manipulative
Bingo cards
Dry-erase boards
Index cards

GROUPING

Individual, partner

TASK DESCRIPTION, DEVELOPMENT, & DISCUSSION

In this task, students will word problems to match given equations.
Comments

The teacher will begin by scaffolding the lesson using a Bingo game. Distribute a card to each student. There are 6 different versions of the card. Thus, you should have multiple winners at once. The teacher will call out each product on the 3 X 3 card in the form of a story problem to further build the student’s understanding of multiplication with real life. She will reinforce to them that multiplication is repeated addition. If they get stuck, use this strategy to figure out the product. The nine factors on the cards are: 36, 20, 18, 28, 35, 16, 21, 24, and 30. The teacher will call out problems for the above products as follows. A task sheet is included.

There are 5 cars.

Each car has 4 tires.

How many tires do they have in all?

The teacher will create a story problem for each product until a winner has been established. While playing, the teacher will use this time to have open discussions about how answers were derived and what strategies they used.

The teacher will then give each student some type of counting manipulative. She will have them create on their desk arrays to compliment the story problems she calls out which will be similar to the aforementioned problems used in the game. However, she will add another sentence which will involve another math operation.

Ex.1
There are 5 cars.

Each car has 4 tires.

3 of the tires are flat.

How many tires are not flat?

Question:
What type of math is being presented now? How would that equation be written? (5 x 4) - 3 = X

The teacher would have the students create arrays and subtract or add manipulatives to solve the equation. Along with that, the students would write the equation for the story problem on a dry erase board and hold the board in the air when the teacher instructs them to do so. This will be done so the teacher can check understanding and all students are engaged. The teacher would give the students more practice problems to build their contextual understanding.

Student Task:

The teacher will break the students into groups of two. The students will be given five index cards. Each index card will have a different equation similar to the ones they had practiced. However, this time, they must work with a partner and create a story problem to match each equation. The teacher should prepare the equations to include an unknown (variable) in different parts of the equation. For example, a x 12 = 36. After completion of the work, the teacher will collect the index cards and redistribute them to other students and have them solve
their classmates’ problems. The more opportunities students are given, the more effective the lesson.

FORMATIVE ASSESSMENT QUESTIONS

• Why is it important to not see math as a single operation?
• What is the relationship between word problems and equations?
• What happens if the equations are not solved in the correct order?

DIFFERENTIATION

Extension
• This lesson can be extended by allowing students to model, using arrays, similar problems with two-digit numbers.

Intervention
• This lesson could be taught in small groups so that more hands-on instruction can be given as needed. Also, during the task, the students could continue to use manipulatives to help create the word problems or even draw a picture.
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<td>36</td>
<td>There are four kids. Each kid has 9 marbles. How many marbles is that?</td>
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<td>20</td>
<td>The gardener has 4 gardens. Each garden has 5 rose bushes. How many rose bushes are there?</td>
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<td>18</td>
<td>Six police officers were patrolling the city. Each one captured 3 bad guys. How many bad guys did they capture?</td>
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<td>28</td>
<td>Seven kids were buying ice cream. They each have four quarters. How many quarters do they have?</td>
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<td>35</td>
<td>There were 5 doctors. Each doctor had 7 patients. How many patients is that?</td>
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<td>16</td>
<td>The pet shop had eight dogs. Each dog has 2 puppies. How many puppies will they have to sell?</td>
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<td>21</td>
<td>There were three teachers. Each teacher had 7 boys each in their class. How many boys were in all three classes?</td>
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<td>24</td>
<td>Three buckets were under an apple tree. Each one could hold 8 apples. What is the largest number of apples that the buckets can hold?</td>
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### 2-Step Word Problem Suggestions

<table>
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<tr>
<th>Problem</th>
<th>Solution</th>
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<tbody>
<tr>
<td>There were 5 parents at the park. Each parent had 3 kids. 6 of the kids were boys. How many were girls?</td>
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<td>The coach had 6 baskets. Each basket contained 7 balls. 12 of them were footballs. How many were not footballs?</td>
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<td>Mary, Luke, Mark, and Isaiah went fishing. They each caught 8 fish. When they got home, their mom had purchased 10 from the local supermarket. How many fish do they have?</td>
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<td>There are 7 fire stations in the city. Each fire station has 5 firemen. During the week, the city hired 8 more. How many firemen do they have in all?</td>
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<td>There were nine students and they each have a pencil box. 7 pencils are inside each one. 28 of the pencils are sharpened. How many are not sharpened?</td>
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PERFORMANCE TASK: WATCH OUR GARDEN GROW!

STANDARDS FOR MATHEMATICAL CONTENT:

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

MCC.3.OA.8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.19

Represent and interpret data.

MCC.3.MD.3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets.

MCC.3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.

   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

MCC.3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

MCC.3.MD.7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show, in a concrete case, that the area of a rectangle with whole-number side lengths $a$ and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTION

- How can area, bar graphs, and multi-step problems be utilized in real-life situations?

MATERIALS

- Inch/centimeter grid paper
- “Watching Our Garden Grow” task sheet

GROUPING

Independent task,

TASK DESCRIPTION, DEVELOPMENT AND DISCUSSION

The students in this culminating task will design a garden and plot different dimensions of area for different flowers while later incorporating the information into a bar graph being sure the y-axis is any interval excluding one. The students will then create one and two-step questions about their garden.

Comments

The teacher will need to point out to the students that each square unit will contain one seed in the garden which will represent one flower. The flowers should be grouped into rectangular plots.

Task Directions

The students could be shown a picture of a flower garden with an assortment of flowers in one garden. Then present the students with the following task:

You want to surprise your mom by planting a flower garden for the Spring. You have measured a plot of land in the backyard that measures 100 square feet. You want to make sure that you plant five of your mom’s favorite flowers. Each square unit in the garden will contain one seed.
- First, design a garden with five different flowers being sure to not exceed 100 square feet. Each flower type should be grouped together.
• Create a bar graph showing the amount of each type of flower you will plant in the garden being sure to make the y-axis any interval of choice excluding one.
• Create 4 questions about your graph. Make sure at least one of them requires more than one operation.

FORMATIVE ASSESSMENT QUESTIONS

• What might your garden look like?
• How did you decide on the area needed to plant each vegetable?
• Is there another way that you could design your garden?
• What’s the connection between your garden and the bar graph?
• How was the garden able to generate questions about the bar graph?
• How would the commutative property change the look of your garden?
• How could the distributive property be applied to plant an additional flower type?

DIFFERENTIATION

Extension
• Have students use colored pencils and on their grid paper, decompose one of the five flower areas and write the equation using the distributive property.

Intervention
• The teacher could give the students manipulatives to use so they could arrange and “see” the flower patterns more clearly.
WATCH OUR GARDEN GROW
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