Illustrative Mathematics

3.NF Comparing Fractions

Alignment 1: 3.NF.A.3

Compare the fractions below. Use the symbols $>$, $=$, or $<$ to record your comparisons. Draw a picture to illustrate your answer.

a. $\frac{2}{6}$ and $\frac{5}{6}$

b. $\frac{1}{2}$ and $\frac{1}{3}$

c. $\frac{3}{6}$ and $\frac{4}{8}$
Commentary:
The purpose of this task is for students to compare fractions using common numerators and common denominators and to recognize equivalent fractions. Students need to represent their answers with both symbols and pictures, so that the visual representation can help students make meaning of the more abstract symbolic representations. Teachers can extend this task to include fractions with larger denominators.

Solution: Comparing fractions

a. $\frac{2}{6} < \frac{5}{6}$

The denominator for both fractions is the same, which tells us that each fraction refers to the same size piece (sixths). If we have only 2 of those pieces, we have fewer pieces than if we have 5 of those pieces. The picture below illustrates this comparison because the shaded area representing $\frac{2}{6}$ is less than the shaded area representing $\frac{5}{6}$:

![Comparison of $\frac{2}{6}$ and $\frac{5}{6}$]

b. $\frac{1}{2} > \frac{1}{3}$

We know that a smaller denominator means that the whole is divided into a fewer number of larger pieces. This means that the pieces of the whole when it is divided into 3 pieces are smaller than the pieces of the same size whole that is only divided into 2 pieces. Since we know we have one of each piece, we know that $\frac{1}{2}$ is greater than $\frac{1}{3}$. The picture below illustrates this comparison because the shaded area representing $\frac{1}{2}$ is greater than the shaded area representing $\frac{1}{3}$:

![Comparison of $\frac{1}{2}$ and $\frac{1}{3}$]

c. $\frac{3}{6} = \frac{4}{8}$

$\frac{3}{6}$ is equal to $\frac{4}{8}$ because these two fractions are both equivalent to the fraction $\frac{1}{2}$. The picture below illustrates this comparison:

![Comparison of $\frac{3}{6}$ and $\frac{4}{8}$]
because the shaded area representing $\frac{3}{6}$ is equal to the shaded area representing $\frac{4}{8}$ and both of these shaded areas represent $\frac{1}{2}$ of the whole:

\[
\begin{align*}
\frac{3}{6} & = \frac{1 \times 3}{2 \times 3} = \frac{1}{2} \\
\frac{4}{8} & = \frac{1 \times 4}{2 \times 4} = \frac{1}{2}
\end{align*}
\]

If we divide the strip above into 6 equal pieces and shade 3 of them, representing $\frac{3}{6}$, we can see that we have shaded $\frac{1}{2}$ of the pieces. This shows us that $\frac{3}{6} = \frac{1}{2}$. If we divide the strip above into 8 equal pieces and shade 4 of them, representing $\frac{4}{8}$, we can see that we have also shaded $\frac{1}{2}$ of the pieces. This shows us that $\frac{4}{8} = \frac{1}{2}$. The picture below shows a different illustration of this comparison:

\[
\begin{align*}
\frac{3}{6} & = \frac{1 \times 3}{2 \times 3} = \frac{1}{2} \\
\frac{4}{8} & = \frac{1 \times 4}{2 \times 4} = \frac{1}{2}
\end{align*}
\]

This is how we might show that these fractions are both equal to $\frac{1}{2}$ using symbols:

\[
\begin{align*}
\frac{3}{6} & = \frac{1 \times 3}{2 \times 3} = \frac{1}{2} \\
\frac{4}{8} & = \frac{1 \times 4}{2 \times 4} = \frac{1}{2}
\end{align*}
\]